

ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI

M. Sc., MATHEMATICS

CIA - II EXAMINATIONS, MARCH - 2020

II - YEAR

SEMESTER - IV

P8MS4001

COMPLEX ANALYSIS - II

Time : 3 Hours

Max Marks : 75

SECTION A ($5 \times 6 = 30$ Marks)

Answer ALL the questions

- 1.(a) If $\{P_n\}$ is an ascending sequence of prime numbers and if

$$\sigma = \operatorname{Re}(s) > 1, \text{ prove that } \frac{1}{\zeta(s)} = \prod_{n=1}^{\infty} (1 - p_n^{-s})$$

(OR)

- (b) Prove that a locally bounded family of analytic functions has locally bounded derivatives.

- 2.(a) State and establish the Harnack's principle.

(OR)

- (b) Prove that a family \mathfrak{F} is totally bounded iff every compact subset $E \subset \Omega$ and every $\epsilon > 0$, there exist $f_1, f_2, \dots, f_n \in \mathfrak{F}$ such that if $f \in \mathfrak{F}$, then $d(f, f_j) < \epsilon$ for some f_j .

- 3.(a) Prove that the zeros a_1, a_2, \dots, a_n and poles b_1, b_2, \dots, b_n of an elliptic function satisfy

$$a_1 + a_2 + \dots + a_n \equiv b_1 + b_2 + \dots + b_n \pmod{M}.$$

(OR)

- (b) Show that a non-constant elliptic function has equally many poles as it has zeros.

- 4.(a) Construct the *Weierstrass* \wp -function.

(OR)

(b) Prove that $\wp(z) - \wp(u) = \frac{\sigma(z-u)\sigma(z+u)}{[\sigma(z)]^2[\sigma(u)]^2}$

5.(a) Explain in detail *Germes* and *Sheaves*.

(OR)

(b) Prove that two analytic continuations $\bar{\gamma}_1$ and $\bar{\gamma}_2$ of a global analytic function \mathbf{f} along the same arc γ are either identical, or $\bar{\gamma}_1(t) \neq \bar{\gamma}_2(t)$ for all t .

SECTION B ($3 \times 15 = 45$ Marks)

Answer any Three questions

6. State and prove *Arzela-Ascoli's* theorem.

7. Derive Schwarz-Christoffel formula.

8. Prove that there exists a basis (ω_1, ω_2) such that the ratio $\tau = \frac{\omega_2}{\omega_1}$ satisfies the following conditions.

(a) $\text{Im } \tau > 0$

(b) $-\frac{1}{2} < \text{Re } \tau \leq \frac{1}{2}$

(c) $|\tau| \geq 1$

(d) $\text{Re } \tau \geq 0$ if $|\tau| = 1$

Also prove the uniqueness of ratio τ , which is determined by the above four conditions.

9. Show that any even elliptic function with periods ω_1, ω_2 can be expressed in the form $c \prod_{k=1}^n \frac{\wp(z) - \wp(a_k)}{\wp(z) - \wp(b_k)}$ (c - constant), provided that 0 is neither a zero nor a pole.

10. State and prove the *Monodromy* theorem.

Dr. STH (30 Copies)

Class : **IIM.Sc. (Mathematics)**
P8MS4002

Semester : **IV**

Subject Code :

Subject Title: **FUNCTIONAL ANALYSIS**

SECTION – A (5× 6=30 MARKS)

1. (a) State and prove Minkowski's inequality.

(OR)

- (b) If N and N' are normed linear spaces, then prove that the set $B(N, N')$ of all continuous linear transformations with respect to the norm $\|T\| = \sup\{\|Tx\|; \|x\| \leq 1\}$ is a normed linear space. Also prove that if N' is Banach Space then $B(N, N')$ is also a Banach Space.

2. (a) State and prove Closed graph theorem.

(OR)

- (b) State and prove Schwarz inequality

3. (a) If A_1 and A_2 are self-adjoint operators on H , then prove that their product $A_1 A_2$ is self-adjoint iff $A_1 A_2 = A_2 A_1$

(OR)

- (b) If T is an operator on H for which $(Tx, x) = 0$ for all x , then prove that $T = 0$.

4. (a) Prove that $\sigma(x)$ is non empty. (OR)

- (b) Prove that the mapping $x \rightarrow x^{-1}$ of G into G is continuous and is therefore a homeomorphism of G onto itself.

5. (a) Prove that the maximal ideal space \mathcal{M} is a compact Hausdorff space.

(OR)

- (b) If A is self-adjoint, then prove that \hat{A} is dense in $C(\mathcal{M})$.

SECTION – B (3× 15=45) Answer Any THREE Questions

6. State and prove Hahn-Banach Theorem
7. State and prove Open Mapping theorem.
8. Let H be a Hilbert space, and let f be an arbitrary functional in H^* then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H .
9. If I is a proper closed two-sided ideal in A , then prove that the quotient algebra A/I is a Banach algebra.
10. State and prove Gelfand representation theorem.

ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI
CIA II, March -2020

Class II M.Sc. Mathematics

Subject Code:P8MS4003

Max. Marks:75

Subject Name: Mathematical Statistics

Time:3 -Hours

SECTION-A (5 X 6= 30)

Answer all Questions

1. (a)Derive the distribution of arithmetic mean of normally distributed random variables.
(OR)

(b) Derive the F distribution

2. (a)State and prove the Kolmogorov theorem.

(OR)

- (b)Test whether is there any relationship between sex and preference of colour.

(Given $x_2=0.05$, $x_1=3.84$)

Colour/Sex	Male	Female
Red	10	40
White	70	30
Green	30	20

3. (a)Explain the sufficiency of an Estimation and give an example.

(OR)

(b)State and prove the Blackwell theorem.

4. (a)Explain the analysis of variance in one-way classification.

(OR)

(b)Let X follow normaldistribution where mean and standard deviation are unknown.Test the hypothesis $H_0:m=m_0$ against the alternative $H_1:m=m_1$.

5. (a)Explain the Sequential Probability Ratio test.

(OR)

(b)The random variable X is normally distributed.Apply SPRT to test $H_0:\sigma=\sigma_0$ against $H_1:\sigma=\sigma_1$.

Section –B (3×15=45Marks)

Answer any THREE questions

6. Define and derive studenttdistribution.

7. Define a contingency table with usual notations. Prove that for (r x s) contingency table

$$\chi^2 = n \sum_{i=1}^r \sum_{k=1}^s \frac{\left[n_{ik} - \frac{n_{i.}n_{.k}}{n} \right]^2}{n_{i.}n_{.k}}$$

8. State and prove the Rao-Cramer's inequality.

9. State and prove Fisher Lemma.

10. Apply the SPRT to test a hypothesis $H_0:P = P_0$ against $H_1:P = P_1$ of the parameter P of a zero one distribution.

ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI
CIA –II (OCTOBER 2020)

Time: 3Hrs

Max.Mark:75

Class: II.M.Sc Maths

Semester- IV Sub.code: P8MS4004

FLUID DYNAMICS

SECTION-A(5 x 6 = 30Marks) Answer All the questions

1. (a) Derive the equation of continuity.

OR

- (b) Discuss the local and particle rates of change.

2. (a) Show that at any point P of a moving inviscid fluid the pressure p is the same in all directions.

OR

- (b) Explain: (a) The Pitot tube (b) Venturi tube.

3. (a) Explain Doublet in a uniform stream.

OR

- (b) Discuss Stoke's stream function.

4. (a) Find the equation of the streamlines due to uniform line source of strength m through the points A(-c,0), B(c,0) and a uniform line sink of strength 2m through the origin.

OR

- (b) Discuss the flow for which $W = Z^2$

5. (a) Discuss Translational motion of Fluid element.

OR

- (b) Discuss the coefficient of viscosity and Laminar flow.

SECTION-B(3 x 15 = 45 Marks)

Answer any Three questions

6. Test whether the motion specified by $\bar{q} = \frac{k^2(\bar{x}_j - \bar{y}_i)}{x^2 + y^2}$ (k = constant) is a possible motion for an incompressible fluid. If so determine the equation of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.
7. Derive the Bernoulli's equation.
8. Discuss (a) Uniform stream (b) simple source
(c) Doublet at o, axis along \overline{OZ}
(d) Uniform line source along \overline{OZ}
9. Discuss the flow due to a uniform line doublet at O of strength μ per unit length, its axis being \overline{OX}
10. Derive the Navier – Stokes equation of motion of a viscous fluid.

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Dr. R.S

ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI

CIA TEST II, March-2020

Class:II M.Sc (Mathematics)

Maximum:75 Marks

Subject:NUMBER THEORY AND CRYPTOGRAPHY Duration:3 Hours

Subject Code: P8MSEP41

SECTION: A(5x 6 = 30 Marks)

Answer ALL the questions

1. (a) Find an upper bound for the number of bit operations required to compute $\binom{n}{m}$.

(Or)

- (b) Convert $\pi = 3.1415296 \dots$ to the base 2.

2. (a) Explain Digraph transformation.

(Or)

- (b) Find the inverse of $A = \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \in M_2(\mathbb{Z} / 26 \mathbb{Z})$.

3. (a) Show that $G^2 = (-1)^{q-1/2} q$.

(Or)

- (b) Prove that $\left(\frac{a}{p}\right) \equiv a^{(p-1)/2} \pmod{p}$.

4. (a) Explain the ElGamal crypto-system.

(Or)

- (b) Examine whether the following sequence (2,3,7, 20, 35, 69) and the volume $V= 45$ is super increasing Knapsack problem and also find the solution.

5. (a) Explain the continued fraction factor algorithm.

(Or)

- (b) Factor 200819 using Fermat factorization.

SECTION: B(3x 15 = 45 Marks)
Answer any **THREE** questions

- 6 (a) Explain the Euclidean algorithm for finding the g.c.d of a and b, where $a > b$.
(b) State and prove Fermat's little theorem.
7. Suppose we know that our adversary is using an enciphering matrix A in the 26-letter alphabet. We intercept the cipher text "WKNCHSSJH" and We know that the first word is "GIVE". Find the deciphering matrix A and write the message.
8. State and prove Law of Quadratic reciprocity.
9. Explain RSA Crypto system with an example.
- 10.If n is strong pseudo-prime to the base b, then prove that it is an Euler – Pseudo prime to the base b

(Dr.STH& Dr. AZ)

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