# ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI CIA TEST II, March 2020 

I M.Sc. MATHEMATICS
Time : 3 hours
ALGEBRA II(P8MS2001)
Max : 75 marks

## SECTION A (5 x $6=30$ Marks) <br> Answer ALL the Questions.

1. (a)If L is an algebraic extension of K and if K is an algebraic extension of F , prove that $L$ is an algebraic extension of $F$.
(OR)
(b) If $\mathrm{a}, \mathrm{b}$ in K are algebraic over F , prove that $\mathrm{a} \pm \mathrm{b}$, ab , and $\mathrm{a} / \mathrm{b}$ if $\mathrm{b} \neq 0$ are all algebraic over $F$.
2. (a)State and prove Remainder theorem.
(OR)
(b) Let $f(x) \in F[x]$ be of any degree $n \geq 1$. Prove that there is an extension $E$ of $F$ of degree at most $n$ ! in which $f(x)$ has $n$ roots.
3. (a) If K is a finite extension of F , prove that $\mathrm{G}(\mathrm{K}, \mathrm{F})$ is a finite group and its order, $\mathrm{O}(\mathrm{G}(\mathrm{K}, \mathrm{F}))$ satisfies $\mathrm{O}(\mathrm{G}(\mathrm{K}, \mathrm{F})) \leq[\mathrm{K}, \mathrm{F}]$
(OR)
(b) Prove that the fixed field of G is a subfield of K .
4. (a)Let F be a field with q elements and suppose that $\mathrm{F} \subseteq \mathrm{K}$, where K is also a finite field. Prove that $K$ has $q^{n}$ elements, where $n=[K: F]$.
(OR)
(b) Prove that any two finite fields having same number of elements are isomorphic.
5. (a)If $\mathrm{a} \in \mathrm{H}$, prove that $\mathrm{a}^{-1} \in \mathrm{H}$ if and only if $\mathrm{N}(\mathrm{a})=1$.
(OR)
(b) Define a Solvable group and give an example.

## SECTION B ( $3 \times 15=45$ Marks) <br> Answer any THREE Questions.

6. Prove that element $\mathrm{a} \in \mathrm{K}$ is algebraic over F if and only if $\mathrm{F}(\mathrm{a})$ is a finite extension of F .
7. Show that a polynomial of degree n over a field can have atmost n roots in any extension field.
8. State and prove the fundamental theorem of Galois theory.
9. State and prove Wedderburn theorem.
10. State and Prove Frobenius theorem.

ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI
CIA TEST II, MARCH 2020
Time: 3 Hours
Max : 75 Marks
M.Sc. MATHEMATICS

Semester II
P8 MS 2002

## REAL ANALYSIS - II

## SECTION A (5 x $6=30$ Marks)

Answer ALL the Questions
1 (a) State and Prove Riesz-Fischer Theorem.
(OR)
(b) State and prove Riemann-Lebesgue lemma.

2 (a) State and prove Taylor's formula.
(OR)
(b) Let $f: R^{2} \rightarrow R^{3}$ be defined by the equation

$$
f(x, y)=(\sin x \cos y, \sin x \sin y, \cos x \cos y) .
$$

Determine the Jacobian matrix $D f(x, y)$.
3 (a) State and prove the Second Derivative Test for extrema.
(OR)
(b) A quadric surface with center at the origin has the equation

$$
A x^{2}+B y^{2}+C z^{2}+2 D y z+2 E z x+2 F x y=1 .
$$

Find the length of its semi-axes.
4 (a) If $E_{1}$ and $E_{2}$ are subsets of $[a, b]$, then prove that

$$
\begin{align*}
& \bar{m} E_{1}+\bar{m} E_{2} \leq \bar{m}\left(E_{1} \cup E_{2}\right)+\bar{m}\left(E_{1} \cap E_{2}\right) \text { and } \\
& \underline{m} E_{1}+\underline{m} E_{2} \geq \underline{m}\left(E_{1} \cup E_{2}\right)+\underline{m}\left(E_{1} \cap E_{2}\right) . \tag{OR}
\end{align*}
$$

(b) Let $f$ be a bounded function on $[a, b]$. Then prove that every upper sum for $f$ is greater than or equal to every lower sum for $f$. That is if $P$ and $Q$ are any two measurable partitions of $[a, b]$ then $U[f ; P] \geq L[f ; Q]$.

5 (a) State and Prove Lebesgue Dominated Convergence Theorem.
(OR)
(b) Derive Schwarz inequality.
6. State and prove Jordan's theorem.
7. If both the partial derivatives $D_{r} f$ and $D_{1} f$ exist in an $n$-ball $B(c ; \delta)$ and if both are differentiable at $c$, then prove that $D_{r, k} f(c)=D_{k, r} f(c)$
8. State and prove the Inverse Function Theorem.
9. If $G_{1}$ and $G_{2}$ are open subsets of $[a, b]$ then, prove that

$$
\left|G_{1}\right|+\left|G_{2}\right|=\left|G_{1} \cup G_{2}\right|+\left|G_{1} \cap G_{2}\right|
$$

10. Let $f$ and $g$ be non-negative valued functions on $[a, b]$. If $f, g \in \mathfrak{L}[a, b]$ then prove that $f+g \in \mathfrak{L}[a, b]$ and

$$
\int_{a}^{b} f+g=\int_{a}^{b} f+\int_{a}^{b} g
$$

Also, prove that $f-g \in \mathfrak{L}[a, b]$ and

$$
\int_{a}^{b} f-g=\int_{a}^{b} f-\int_{a}^{b} g .
$$

# ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI - 2 

CIA- II, MARCH - 2020

## Class: I M.Sc (Mathematics)

Maximum : 75 Marks
Subject : Partial Differential Equations (P8MS2003)
Duration : $\mathbf{3}$ Hours
SECTION : A ( $5 \times 6=30$ Marks )
Answer ALL the questions

1. (a)Find the general integral of the $\operatorname{PDE} y^{2} p-x y q=x(z-2 y)$
[OR]
(b) Find the general integral of the PDE: $\left(x^{2}-y z\right) p+\left(y^{2}-z x\right) q=z^{2}-x y$.
2. (a)Obtain the canonical form of $x^{2} u_{x x}-2 x y u_{x y}+y^{2} u_{y y}=e^{x}$
[OR]
(b) Reduce the following equation to canonical form $\left(1+x^{2}\right) u_{x x}+\left(1+y^{2}\right) u_{y y}+x u_{x}+y u_{y}=0$
3. (a)Derive Poisson equation.
[OR]
(b) Derive Laplace equation.
4. (a) Derive Fourier heat conduction equation.
[OR]
(b) The ends A and B of a rod, 10 cm in length, are kept at temperatures $0^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$ until the steady state condition prevails. Suddenly the temperature at the end A is increased to $20^{\circ} \mathrm{C}$, and the end $B$ is decreased to $60^{\circ} \mathrm{C}$. Find the temperature distribution in the rod at time ' $t$ '.
5. (a) Derive one-dimensional wave equation.
[OR]
(b) Prove that the solution to the wave equation is unique.

SECTION : B ( $3 \times 15=45$ Marks )
Answer any THREE the questions
6. Show that the following PDEs : $x p-y q=x$ and $x^{2} p+q=x z$ are compatible andhence find their solution.
7. Reduce the following equation to a canonical form and hence solve $u_{x x}-2 \sin x u_{x y}-\cos ^{2} x u_{y y}-\cos x u_{y}=0$.
8. Obtain the solution of Laplace equation in sphericalcoordinates.
9. Obtain the solution of the diffusion equations in cylindricalcoordinates.
10. State and prove Duhamel's principle.

## ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI

CIA TEST - I March - 2020
P8MS2004 - ADVANCED NUMERICAL ANALYSIS Time : 3 Hours

Max. Marks : 75

## Section-A ( $5 \times 6=30$ Marks) <br> Answer any ALL questions

1. (a) Prove that Newton-Raphson method has second order convergence.

## (Or)

(b) Find all the roots of the polynomial using the Graeffe's root squaring method.
2. (a) Estimate the eigenvalues of the matrix $A=\left[\begin{array}{ccc}1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1\end{array}\right]$ using the Gerschgorin bounds.
(b) Find the largest eigenvalue in modulus and the corresponding eigen vector of the matrix $A=\left[\begin{array}{ccc}-15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2\end{array}\right]$ using the power method.
3. (a) Find $f(0.25,0.75)$, using linear interpolation for the following

| $y / x$ | 0 | 1 |
| :--- | :--- | :--- |
| 0 | 1 | 1.414214 |
| 1 | 1.732051 | 2 |
| $($ Or $)$ |  |  |

(b) Obtain the Chebyshev linear polynomial approximation to the function $f(x)=x^{3}$ on $[0,1]$.
4. (a) Derive the first derivative of second order using method of undetermined coefficients.
(b) Find the Jacobian matrix for the system of equations $f_{1}(x, y)=x^{2}+y^{2}-x=0 f_{2}(x, y)=x^{2}-y^{2}-y=0$ at the point ( 1,1 ), using the methods $\left(\frac{\partial f}{\partial x}\right)_{\left(x_{i}, y_{j}\right)}=\frac{f_{i+1, j}-f_{i-1, j}}{2 h}\left(\frac{\partial f}{\partial y}\right)_{\left(x_{i, y}, y_{j}\right)}=\frac{f_{i, j+1}-f_{i, j-1}}{2 k}$ with $h=k=1$.
5. (a) Use Euler's method to solve numerically the IVP $u^{\prime}=-2 t u^{2}, u(0)=1$ with $h=0.1$ on the interval $[0,1]$.

## (Or)

(b) Explain the Taylor series method.

$$
\begin{aligned}
& \text { Section - B }(3 \times 15=75 \text { Marks }) \\
& \text { Answer any THREE questions }
\end{aligned}
$$

6. Perform two iterations with the Muller method for the equation $\log x-x+3=0, x_{0}=\frac{1}{4}, x_{1}=\frac{1}{2}, x_{2}=1$.
7. Find all the eigen values and eigen vectors of the matrix $\left[\begin{array}{ccc}1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1\end{array}\right]$ using Jacobi method for symmetric matrices.
8. Given the following values of $f(x)$ and $f^{\prime}(x)$

| $x$ | $f(x)$ | $f^{\prime}(x)$ |
| :---: | :---: | :---: |
| -1 | 1 | -5 |
| 0 | 1 | 1 |
| 1 | 3 | 7 |

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.
9. Evaluate the double integral $\int_{11}^{2} \int_{1}^{2} \frac{d x d y}{x+y}$ using the trapezoidal rule with $h=k=0.5$ and $h=k=0.25$.
10. Solve the IVP $u^{\prime}=-2 t u^{2}, u(0)=1$ with $h=0.2$ on the interval $[0,1]$ using fourth order classical Runge-Kutta method.

I M.Sc., (Mathematics)Subject : (Elective) Operations Research
Time: 3 hours
Max: 75 marks
Section - A ( $5 \times 6=30$ marks )
Answer All questions

1. (a) Define the following (i)Regret Criterion (ii) Equal probability criterion (OR)
(b) Discuss the difference between decision making under certainty and under risk
2. (a) What are the differences between CPM and PERT.(OR)
(b) Explain time cost trade off procedure.
3. (a) Derive Harris lot size formula (OR)
(b) A manufacturer has to supply his customer with 600 units of his production per year. Shortages are not allowed and the shortage cost amount to Rs $0.60 /$ unit/year. The set up cost per run is Rs 80 . Find the optimum lot size and the minimum average cost.
4. (a) In a railway marshalling yard, goods trains arrive at a rate of 30 train/day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, find
i. Expected queue size
ii. Probability that the queue size exceeds(OR)
(b) Consider a single server queuing system with Poisson input, exponential service times. Suppose that mean arrival rate is 3calling units per hour, the expected service time is 0.25 hours and the maximum permissible calling units in the system is two. Derive the steady state probability distribution of the number of calling units in the system and then calculate the expected number with system.
5. (a) A pipe line is due for repairs. It will cost Rs 10,000 and lasts for three years. Alternatively, new pipe line can be laid at cost of Rs 30,000 and lasts for 10 years. Assuming cost of capital to be 10 percentand ignoring salvage value, which alternative should be chosen?(OR)
(b) A firm is considering replacement of a machine whose cost price is Rs 12,200 and the scrap value is Rs 200. The running cost are found from experience to be as follows

| Year : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Running cost : | 200 | 500 | 800 | 1200 | 1800 | 2500 | 3200 | 4000 |

When should the machine be replaced?

## Section - B ( $3 \times 15=45$ marks ) <br> Answer any three questions

6. The probability of the demand for lorries for hiring on any day in a district is as follows :

| No. of lorries <br> demanded : | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Probability : | 0.1 | 0.2 | 0.3 | 0.2 | 0.2 |

Lorries have fixed cost of Rs 90 each day to keep the daily hire charge Rs 200. If the lorry hire company owns 4 lorries, what is the daily expectation? If the company is about to go into business and currently has no lorries, how many lorries should it but?
7. A Project is represented by the following data

| Activity | $:$ | $1-2$ | $1-3$ | $1-4$ | $2-5$ | $3-5$ | $4-6$ | $5-6$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| In weeks $t_{0}$ | $:$ | 1 | 1 | 2 | 1 | 2 | 2 | 3 |
| In weeks $t_{m}$ | $:$ | 1 | 4 | 2 | 1 | 5 | 5 | 6 |
| In weeks $t_{p}$ | $:$ | 7 | 7 | 8 | 1 | 14 | 8 | 15 |

(i) Draw the network and Find the expected duration and variance for each activity.
(ii) What is expected project length?
8. The production department for a company requires 3600 kg of raw materials for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs 36 and the cost of carrying inventory is 25 per cent of the investment in the inventories. The price is Rs 10 per kg . The purchase manager wishes to determine an ordering policy for raw material.
9. Derive the differencial equation for the queuing model $(M / M / 1):(\mathrm{N} / \mathrm{FCFS})$
10. The following mortality rates have been observed for a certain type of fuse :

| Week | $:$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Per cent failing by end of the week | $:$ | 5 | 15 | 35 | 57 | 100 |

There are 1000 fuses in use and it costs Rs 5 to replace an individual fuse. If all fuses were replaced simultaneously it would cost Rs 1.25 per fuse. It is proposed to replace all fuses at fixed intervals of time whether or not they have burnt out and to continue replacing burnt out fuses as they fail. Which is the best replacement policy.

ISLAMIAH COLLEGE [AUTONOMOUS], VANIYAMBADI CIA TEST II - MARCH 2020

Time: 3 Hrs. Class:

Sem: II Sub. Code: P8HR2001

## HUMAN RIGHTS

 PART - A ( $5 \times 6=30$ Marks)
## Answer ALL questions

1. (a) Discuss about the important definitions of Human Rights.
(Or)
(b) Highlight on the important theories of Human Rights.
2. (a) Write about the Civil and Political Rights enshrined in the International Covenant.
(Or)
(b)Describe about the Economic, Social and Cultural Rights in the International Covenant.
3. (a) Write about the powers and Functions of the United Nations High Commission for Refugees.
(Or)
(b) Discuss about the role of U.N.O. in Safeguarding Human Rights.
4. (a) Write about the Helsinki Process.
(Or)
(b)Discuss about the monitoring of Human Rights in Europe.
5. (a) Write a note on the Directive Principles of State policy in Indian Constitution.
(Or)
(b) What are the powers and Functions of National Human Rights Commission.

> PART - B $(3 \times 15=45$ MARKS $)$
> Answer any THREE of the following
6. Trace the historical development of Human Rights.
7. Highlight on the significance of Universal Declaration of Human Rights.
8. Discuss about the Powers, Functions and Duties of United Nations High Commission for Human Rights.
9. Discuss about the role of Amnesty International in Safeguarding Human Rights.
10. Describe in detail regarding the Fundamental Rights enshrined in the Indian Constitution.

