ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI
B. Sc., MATHEMATICS

| CIA - II EXAMINATIONS, MARCH - 2020 |  |  |
| :---: | :---: | :---: |
| III - YEAR | SEMESTER - VI | U5MS6001 |
| MODERN ALGEBRA - II |  |  |

Time: 3 Hours Max Marks : 75

SECTION A $(10 \times 2=20$ Marks $)$
Answer ALL the questions

1. Define vector space.
2. Define homomorphism in a vector space.
3. Define dual space.
4. Prove that $\|\alpha u\|=|\alpha|\|u\|, \quad u \in V, \alpha \in F$.
5. Define linear transformation.
6. Define characteristic root of a linear transformation.
7. When a linear transformation is said to be similar?
8. Define invariant subspace.
9. Define trace.
10. Define Skew-Hermitian matrix.

## SECTION B ( $5 \times 5=25$ Marks)

Answer ALL the questions
11.(a) If $v_{1}, v_{2}, \ldots, v_{n}$ are in $V$, then prove that either they are linearly independent or some $v_{k}$ is a linear combination of the preceeding ones. (OR)
(b) If $v_{1}, v_{2}, \ldots, v_{n}$ is a basis of $V$ over $F$ and if $w_{1}, w_{2}, \ldots, w_{m}$ in $V$ are linearky independent over $F$, then prove that $m \leq n$.
12.(a) If $V$ is finite dimensional, prove that $\psi$ is an isomorphism of $V$ onto $\widehat{\widehat{V}}$. (OR)
(b) State and prove Schwarz inequality.
13.(a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then show that for any polynomial $q(x) \in F[x], q(\lambda)$ is a characteristic root of $q(T)$.
(OR)
(b) Prove that an element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0 \in V$ such that $v T=\lambda v$
14.(a) Let $V$ be a vector space of all polynomials over $F$ of degree 3 or less and let $D$ be the differential operator defined by
$\left(\alpha_{0}+\alpha_{1} x+\alpha_{2} x^{2}+\alpha_{3} x^{3}\right) D=\alpha_{1}+2 \alpha_{2} x+3 \alpha_{3} x^{2}$. Find the matrix of $D$ in the basis $1,1+x, 1+x^{2}, 1+x^{3}$.
(OR)
(b) If $V$ is $n$-dimensional over $F$ and if $T \in A(V)$ has all its characteristic roots in $F$, prove that $T$ satisfies a polynomial of degree $n$ over $F$.
15.(a) If $S$ is of characteristic zero and if $S$ and $T$ in $A_{F}(V)$, such that $S T-T S$ commutes with $S$. Prove that $S T-T S$ is nilpotent. (OR)
(b) If $A, B \in F_{n}$ and $\lambda \in F$. Prove the following.
(i) $\operatorname{tr}(A+B)=\operatorname{tr} A+\operatorname{tr} B$.
(ii) $\operatorname{tr}(\lambda A)=\lambda \operatorname{tr} A$
(iii) $\operatorname{tr}(A B)=\operatorname{tr}(B A)$.

## SECTION C ( $3 \times 10=30$ Marks)

Answer Any Three questions
16. If $V$ is finite dimensional and if $W$ is a subspace of $V$. Prove that $W$ is finite dimensional, $\operatorname{dim} W \leq \operatorname{dim} V$ and $\operatorname{dim}\left(\frac{V}{W}\right)=\operatorname{dim} V-\operatorname{dim} W$.
17. Explain Gram-Schmidt orthogonalization process.
18. If $V$ is finite dimensional vector space over $F$. Prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for $T$ is not zero.
19. If $T \in A(V)$ has all its characteristic roots in $F$, then prove that there is a basis of $V$ in which the matrix of $T$ is triangular.
20. If $F$ is a field of characteristic zero and if $T \in A(V)$ is such that $\operatorname{tr} T^{i}=0, i \geq 0$. Prove that $T$ is nilpotent.

## ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI

 CIA -II (March2020)Time: 3 Hrs
Max.Mark:75

## Class: III B.Sc Maths Semester- VI Sub.code: U5MS6002

## Complex Analysis

## Section - A (10 $\times 2$ = 20 Marks $)$

## Answer ALL Questions

1. Define Analytic function.
2. Show that function $f(z)=\bar{z}$ is not analytic.
3. Define Fixed Point.
4. Define Bilinear Transformation
5. Define Line Integral.
6. State Cauchy Goursat Theorem.
7. Expand $e^{z}$ in a Taylor's series about $z=0$.
8. State Laurent's theorem.
9. Define pole of order m .
10. Define singular point.

## Part - B (5 $\times 5=25$ Marks) <br> Answer ALL Questions

11. a)If $f(x)$ is analytic then prove that

$$
\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)|f(z)|^{2}=4\left|f^{\prime}(z)\right|^{2} .
$$

(OR)
b)If $f(z)=u(x, y)+i v(x, y)$ is an analytic function and $u(x, y)=\frac{\sin 2 x}{\cosh y+\cos 2 x}$ find $f(z)$.
12. a)Put the Bilinear Transformation $w=\frac{3 z+4}{z-1}$ in its normal form.
(OR)
b)Discuss the transformation $w=z^{2}$
13. a)State and prove Cauchy's Integral formula.
b)Evaluate $\int_{C} \frac{e^{2 z}}{(z+1)^{4}} d z$ where C is the circle $|z|=2$ by using

Cauchy's integral formula.
14. a)Expand $f(z)=\frac{1}{z}$ about (i) $z=1$ (ii) $z=2$ (iii) $z=i$
(OR)
b) Find the Laurent's series expansion of $f(z)=\frac{1}{z^{2}+3 z+2}$ in the region $1<|z|<2$.
15. a)State and prove Cauchy's Residues theorem.
(OR)
b)Find the residues and poles of $f(z)=\frac{z+1}{z^{2}(z-2)}$
(OR)

## Part - C ( $\mathbf{3 \times 1 0 = 3 0}$ Marks)

## Answer any THREE Questions

16. State and prove necessary condition for analytic function.
17. Find Bilinear transformation which maps the pointsz $=$ $2, i,-2$ into the points $w=1, i,-1$ respectively. Also find the invariant points in this transformation.
18. State and prove Maximum modulus principle.
19. State and prove Taylor's Theorem
20. Prove that $\int_{0}^{\infty} \frac{2 x^{2}-1}{x^{4}+5 x^{2}+4} d x=\frac{\pi}{4}$ using contour integration.

## ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI - 635752

CIA TEST II - March 2020
Time : 3 Hours
Max. Marks : 75
Class :B.Sc., Mathematics Semester : VI Sub. Code :

## U5MS6003

## Subject Name :DYNAMICS

PART - A ( $\mathbf{1 0} \times 2=20$ Marks $)$

## Answer all the questions

1. Define Work and Energy.
2. State the Newton's First Law of Motion.
3. A particle is projected with a velocity of $9.6 \mathrm{~m} / \mathrm{s}$ at an angle of $30^{\circ}$. Find the time of flight.
4. Define Angle of projection and Horizontal Range.
5. Define Impulse.
6. Define Oblique Impact.
7. Define Epoch and Phase of a SHM.
8. Write the formula for the velocity and displacement of a SHM.
9. Define Moment of Inertia.
10. Define the Radius of Gyration.

## PART - B ( $5 \times 5=25$ Marks)

## Answer all the questions

11. (a) Verify the Principle of energy in the case of a freely falling body.

## (or)

(b) State and prove the principle of work-energy
12. (a) Obtain the range on an inclined plane by a projectile.

## (or)

(b) Find the velocity and direction of a shot which passes in a horizontal direction over the top of a wall 100 m away and 50 m high.
13. (a) Discuss the loss of kinetic energy in impact.
(b) Find the velocities of two smooth spheres after a direct impact between them.
14. (a)A particle is moving with S.H.M. and while making an oscillation from one extreme position to the other, its distances from the centre of oscillation at 3 consecutive seconds are $x_{1}, x_{2}, x_{3}$. Prove that the period of oscillation is $\frac{2 \pi}{\cos ^{-1}\left(\frac{x_{1}+x_{3}}{2 x_{2}}\right)}$.
(or)
(b) Discuss the composition of two Simple Harmonic Motions of the same period and in the same straight line.
15. (a)State and prove the Perpendicular Axes theorem.
(or)
(b)Find the Moment of Inertia of a thin uniform rod.

> PART $-C(3 \times 10=30$ Marks $)$
> Answer any THREE questions
16. State and prove the principle of conservation of energy.
17. Show that the path of a projectile is a parabola.
18. Find the velocities of two smooth spheres after a oblique impact between them.
19. Show that the resultant of two simple harmonic motions in the same direction and of equal periodic time, the amplitude of one being twice that of the other and its phase a quarter of a period in advance, is a simple harmonic motion of amplitude $\sqrt{5}$ times that of the first and whose phase in advance of the first by $\frac{\tan ^{-1} 2}{2 \pi}$ of a period.
20. State and prove the Parallel Axes theorem.

## ISLAMIAH COLLEGE (AUTONOMOUS), VANIYAMBADI CIA TEST - II, March - 2020 <br> OPERATIONS RESEARCH - II (U5MS6004)

Time : 3 Hours
Max Marks :75

## Section A (10 $\times 2=20$ Marks) <br> Answer ALL questions

1. What is the meant by activity?
2. Write down the formula of LST
3. Define penalty cost.
4. Write the formula of minimum average cost in inventory.
5. Define sequencing problem.
6. Define Total elapsed time.
7. Define waiting time in the system.
8. Define Queue.
9. What are the two types of replacement policies?
10. Define discount

## Section B (5 $\times 5=25$ Marks)

## Answer ALL questions

11 (a) Explain 3 time estimates in PERT calculation.

## (OR)

(b) Differentiate between CPM and PERT.

12 (a) A company has a demand of 12,000 units /year for an item and it can produce 2,000 items per month. The cost of one setup is Rs. 400 and the holding cost/unit/month is Rs. 0.15. Find the optimum lot size and the total variable yearly cost. (OR)
(b) A manufacturer has to supply his customer with 600 units of his product per year.Shortage is not allowed and the storage cost amount to be Rs. 0.60 per unit /year. The setup cost for one run is Rs. 80. Find the optimum run size and minimum average yearly cost.

13 (a) There are 5 jobs and 2 machines A and B Processing time in hours are given below.

| Job: | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Machine A | 10 | 2 | 18 | 6 | 20 |
| Machine B | 4 | 12 | 14 | 16 | 8 |

(OR)
(b) Solve the sequencing problem

| Task | $A$ | $B$ |  | $D$ | $E$ | $F$ | $G$ | $H$ |  | $I$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M 1$ | 2 | 5 | 4 | 9 | 6 | 3 | 7 | 5 |  | 4 |
| $M 2$ | 6 | 8 | 7 | 4 | 3 | 9 | 3 | 8 |  | 11 |

14 (a) A customer arrives at box office window being named by individual according to poison input process with a mean rate of 30 per hours. A time required to serve a customer as an exponential distribution with mean of 90 seconds. Find the average waiting time of a customer?
(OR)
(b) Cars arrive at a petrol pump, having one petrol unit, in poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find the
(i) average number of cars in the system.
(ii) average waiting time in the queue.
(iii) Average queue length
(iv) The probability that the number of cars in the system is 2.

15 (a) Let the value of the money be $10 \%$ per year and suppose that machine A is replaced after every 3 years whereas machine $B$ is replaced after every six years.

| Year | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine A | 1000 | 200 | 400 | 1000 | 200 | 400 |
| Machine B | 1700 | 100 | 200 | 300 | 400 | 500 |

Determine which machine should be purchased?
(OR)
(b) Let $\mathrm{V}=0.9$ and initial price Rs. 5000 running cost are given in the table below.

| Tears | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Running <br> cost | 400 | 500 | 70 | 1000 | 1300 | 1700 | 2100 |

What would be the optimum replacement interval?

## Section C ( $\mathbf{3} \times \mathbf{1 0}=\mathbf{3 0}$ Marks $)$

Answer any THREE questions
16. Construct the network for the project whose activities are given below. Compute total float, free float and independence float of each activity and hence determine critical path and project duration.

| Activity | $0-1$ | $1-2$ | $1-3$ | $2-4$ | $2-5$ | $3-4$ | $3-6$ | $4-7$ | $5-7$ | $6-7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Duration | 3 | 8 | 12 | 6 | 3 | 3 | 8 | 5 | 3 | 8 |

17. Use graphical method to minimize the time needed to process the following jobs on machine given below.

| Job 1 | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| Sequence of machine of time(in hours) | 4 | 6 | 7 | 3 |
| Job 2 | D | B | A | C |
| Sequence of machine of time(in hours) | 8 | 7 | 4 | 5 |

18. When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows

| Number of mistakes in <br> a page (x) | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No of pages (f) | 275 | 72 | 30 | 7 | 5 | 3 |

19. Western national bank us considering opening a derive in window for customer service management estimates that the customer will arrive for service at rate of 15 per hour. The teller whom to staff the window can service customers at the rate of in every three minutes. Assuming poison arrival and exponential service time.

Find:
(i) Utilization of the teller.
(ii)Average number in waiting line.
(iii) Average waiting time in the line.
(iv) Average waiting time in the system.

20 The following mortality rate has been observed for a certain type of fuse.

| Fuse failure / <br> week: | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \% of failure at <br> the end of <br> week: | 5 | 15 | 35 | 75 | 100 |

There are 1000 fuses in use and it cost Rs. 5 to replace individual fuse. If the all fuses are replaced simultaneously, it would cost Rs.1.25 per fuse. At what interval the group replacement should be made and suggest your answer?

# ISLAMIAH COLLEGE(AUTONOMOUS), VANIYAMBADI CIA-II MARCH-2020 

## III-B.ScMathematics

Sub: FUZZY SET THEORY
Code: U5MS6005
Time: 3 hours

## SECTION - A(10 $\times \mathbf{2}=\mathbf{2 0}$ Marks) <br> Answer all the Questions

1. Define the concept of a fuzzy set.
2. Define the following for the fuzzy set $A$
a) Support
b) Height
3. State second decomposition theorem.
4. Define the extension principle for fuzzy sets.
5. Define fuzzy complement.
6. Give the example of t-norms in fuzzy intersection.
7. State the characterization theorem of $t$-conorms.
8. Define an aggregation operation.
9. Find the value of a) $[2,5]+[1,3]$ b) $[2,5]-[1,3]$
10. Write the four basic arithmetic operations on fuzzy numbers.

## SECTION-B ( $5 \times 5=\mathbf{2 5}$ Marks)

Answer all the Questions
11. (a) Explain why the law of contradiction and the law of exclusive middle are violated in fuzzy set theory under the standard fuzzy sets operations. What is the significance of this?
(OR)
(b) State the important features of new paradigm.
12. (a) Let $A, B \in \mathscr{F}(X)$ and for all $\alpha, \beta \in[0,1]$, prove that $\quad{ }^{\alpha}(\bar{A})={ }^{(1-\alpha)^{+}} \bar{A}$
(OR)
(b) Let $A \in \mathfrak{F}(X)$. Then, Prove that

$$
{ }^{\alpha} A=\bigcap_{\beta<\alpha}{ }^{\beta} A=\bigcap_{\beta<\alpha}{ }^{\beta+} A
$$

13. (a)If a complement $c$ has an equilibrium $e_{c}$, then prove that ${ }^{d} e_{c}=e_{c}$
(OR)
(b) If $c$ is a continuous fuzzy complement, then prove that $c$ has a unique equilibrium.
14. (a)Prove that the standard fuzzy union is the only idempotent t -conorm.
(OR)
(b) Prove that the triples $\langle\min , \max , c\rangle$ and $\left\langle i_{\min } \quad, u_{\max } \quad, c\right\rangle$ are dual with respect to any fuzzy complement $c$
15. (a) Discuss the Linguistic Variables.

> (OR)
(b) Let $A(x)=\left\{\begin{array}{lc}0 & \text { for } x \leq-1 \text { and } x>3 \\ \frac{(x+1)}{2} & \text { for }-1<x \leq 1 \\ \frac{(3-x)}{2} & \text { for } 1<x \leq 3\end{array}\right.$ and

$$
B(x)=\left\{\begin{array}{cc}
0 & \text { for } x \leq 1 \text { and } x>5 \\
\frac{(x-1)}{2} & \text { for } 1<x \leq 3 \\
\frac{(5-x)}{2} & \text { for } 3<x \leq 5
\end{array}\right.
$$

Find (i) $(A+B)(x)$, (ii) $(A-B)(x)$

## SECTION-C ( $\mathbf{3} \times \mathbf{1 0}=\mathbf{3 0}$ Marks) <br> Answer any three questions

16. Prove that a fuzzy set $A$ on $R$ is convex if and only if $A\left(\lambda x_{1}+(1-\lambda) x_{2}\right) \geq$ $\left.\min \Phi A\left(x_{1}\right), A\left(x_{2}\right)\right]$ for all $x_{1}, x_{2} \in R$ and all $\lambda \in[0,1]$ where min denotes the minimum operator.
17. State and prove First Decomposition theorem.
18. State and prove Second Characterization theorem.
19. Let $\langle i, u, c\rangle$ be a dual triple. Prove that the fuzzy operations $i, u, c$ satisfies the law of excluded middle and the law of contradiction.
20. Let $* \in\{+,-, ., /\}$, and let $A$ and $B$ denote continuous fuzzy numbers.Prove that the fuzzy set $A * B$ defined by $\left.(A * B)(z)=\sup _{\mathrm{z}=\mathrm{x} * \mathrm{y}} \min \mathrm{P} A(x), B(y)\right] \quad$ is a continuous fuzzy number.

# ISLAMIAH COLLEGE (AUTONOMOUS) <br> CIA TEST II, March-2020 

Class: III B.Sc. (Mathematics) Semester: VI
Computational Methods Code: U5MSSB61
Maximum: 75 Marks Subject:
Duration: 3 Hours
PART - A ( $10 \times 2=20$ Marks $)$
ANSWER ALL THE QUESTIONS

1. Find the unit's digit in $684 \times 759 \times 413 \times 676$.
2. Solve $875 \times 125+875 \times 125$.
3. Define Permutations and Combinations.
4. How many different ways can the letter of the word "INDIA" be arranged?
5. Sachin is younger than Rahul by 4 years. If their ages are in the ratio of $7: 9$, then how old is Sachin.
6. At what angle the hour hand and the minute hand of a clock when the time is 3.25.
7. Find the present age of Sam, whose age after 15 years will be 5 times his age 5 years back.
8. How many times in a day, the hands of a clock are straight?
9. Find the cost of Rs $6200,8 \%$ stock at 90.
10. Define Shares and Face Value.

PART - B ( $5 \times 5=25$ Marks)
ANSWER ALL THE QUESTIONS
11. (a) How many natural numbers between 17 and 80 are divisible by 6 ?

Or
(b)
(b) Find the sum of all 2-digit numbers divisible by 3 .
12. (a) In how many different ways can the letter of the word MATHEMATICS be arranged so that the vowels always come together?

## Or

(b) From a group of 6 men and 6 women, 5 persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?
13. (a) Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages?

Or
(b) The present ages of three persons are in the proportions 5:7:8. Eight years ago the sum of their ages were 80 . Find their present ages.
14. (a) At what time between 2 and 3 o'clock will the hands of a clock is together?

Or
(b) Find at what time between 8 and 9 o'clock will the hands of a clock be in the same straight line but not together.
15. (a) Find the cash realized by selling Rs. $2440,9.5 \%$ stock at 4 discounts (brokerage $1 / 4 \%$ ).

> Or
(b) A man buys Rs. 25 shares in a company which pays $9 \%$ dividend. The money invested is such that it gives $10 \%$ on investment. At what price did he buy the shares?

## PART - C ( $\mathbf{3} \mathbf{x} 10=30$ Marks) <br> ANSWER ANYTHREE QUESTIONS

16. (a) Find the HCF and LCM of $0.63,1.05$ and 2.1.
(b) Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in ascending and descending orders.
17. (a) How many words can be formed from the letters of the word 'DIRECTOR'so that the vowels always come together?
(b) In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
18. (a) The present ages of $X$ and $Y$ are in the proportions 5:6 respectively. Seven years hence this proportion will become $6: 7$ respectively. Find their present ages of $X$ and $Y$ in years.
(b) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages were 34 . Find the ages of father and son.
19. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?
20. A man sells Rs. 5000, $12 \%$ stock at 156 and invests the products partly in $8 \%$ stock at 90 and $9 \%$ stock at 108 . He there by increases his income by Rs. 70. How much of the proceedswereinvested in each stock?

Dr.AZ
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